RESEARCH STATEMENT (SHORTER)

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Overview

I study mathematical spaces through the lens of two distinct types of geometry: algebraic and symplectic. The former has its roots in formalising Hamiltonian dynamics, which for example can describe the movement of celestial bodies. The latter, on the other hand, originated in the study of polynomial equations and their zero sets. These two types of geometry are connected by a deep and mysterious duality called mirror symmetry.

A common theme in my research is a type of singular space called an orbifold whose singularities are locally modelled on a flat space which is folded along a set of symmetries. The geometry of orbifolds is intimately related to the representation theory of finite groups. While orbifolds have historically occupied a more central place in algebraic geometry, recent advances in orbifold Floer theory and orbifold Gromov-Witten theory ([9]) have led to a surge of interest in them from the symplectic geometry community.

For a big part of my PhD, I investigated orbifolds through the viewpoint of the homological mirror symmetry (HMS) conjecture which states that certain categorical structures within algebraic and symplectic geometry are intertwined. I focused on a class of varieties called log Calabi-Yau surfaces and showed in [11] how to extend HMS from the smooth case to the orbifold case.

During my investigations into HMS for orbifolds I studied them as algebraic varieties and related them to their symplectic counterparts. However, in current work-in-progress with Elliot Gathercole, we consider orbifolds from the symplectic point of view in order to study an invariant known as symplectic cohomology. We develop new computational tools which we apply to compute the symplectic cohomology of various smoothings of hypersurface singularities. As another sample application of our results, we completely classify all compactifications of $\mathbb{C}^3$ with an orbifold divisor admitting at most one orbifold point.

1. Homological mirror symmetry for log Calabi-Yau surfaces with cyclic quotient singularities

There is a trichotomy in algebraic geometry governed by a quantity known as the first Chern class which, roughly, measures their curvature. Of particular importance are the ones which are flat: these are the Calabi-Yau manifolds, which first gained importance in various models of supersymmetric field theory and string theory. Calabi-Yau manifolds themselves come into two types: the compact ones, which have finite volume, and the open ones, which are unbounded. Mathematically and physically, it is an important problem to compactify the unbounded Calabi-Yau manifolds, which can be thought of as filling in the gaps of infinite area, or alternatively bounding the manifold with a sort of wall called a divisor. In complex dimension two, such compactifications are called log Calabi-Yau surfaces.

The same open Calabi-Yau manifold U can have many different compactifications, some of which can have orbifold singularities. Moreover, somewhat surprisingly, most of the geometry of the compact surface is contained entirely in the compactifying divisor. The upshot is that one can understand a 2-dimensional surface by carefully studying how the compactifying divisor sits inside of it.

Mirror symmetry is a duality in mathematics and physics which exchanges Calabi-Yau manifolds: the principle is that two dual spaces, called the A-model and B-model, lead to the exact same physical theory.

This duality is compatible with compactifications: if U is mirror dual to $\check{U}$ and (Y, D) compactifies $U = Y \setminus D$, then there is an associated a function f_Y on $\check{U}$. This function defines what is called a Landau-Ginzburg model.

To make mirror symmetry a rigorous mathematical statement, one associates structures to these spaces: on the B-side, there is the derived category of coherent sheaves $D^b(Y)$ and on the A-side is the category of vanishing cycles $\mathcal{F}(W, w)$. The homological mirror symmetry conjecture predicts that mirror spaces carry the same underlying categorical structure: $D^b(Y) \simeq \mathcal{F}(W, w)$. This conjecture has been verified by Hacking and Keating [3] for smooth log Calabi-Yau surfaces, building on previous work of Gross-Hacking-Keel. Moreover, it is proved in [7] for a special class of singular spaces known as nodal stacky curves which are the compactifying divisors in an orbifold log CY pair. In my paper, I extend this conjecture to include the case of log Calabi-Yau surfaces with cyclic quotient singularities. This amounts to a reasonable jump in complexity: the mirror to a smooth log CY is a Lefschetz fibration whose fiber is a torus (genus one), whereas the mirror to an orbifold log CY might have fibers of arbitrarily high genus.

Theorem 1.1. *([11]) Suppose $(\mathcal{X}, \mathcal{D})$ is an orbifold log Calabi-Yau compactification of an open Calabi-Yau variety U, with $\operatorname{Sing}\mathcal{X} = \operatorname{Sing}\mathcal{D}$. Let (Y, D) be the minimal resolution of the coarse space of $\mathcal{X}$. Provided that Y is at the large complex structure, there are equivalences (horizontal) and fully faithful inclusions (vertical) fitting into the following diagram:*

$$\begin{tikzcd} D^b(Y) \arrow[r, "\simeq"] \arrow[d, hook] & \mathcal{F}(\check{U}, w_Y) \arrow[d, hook] & \operatorname{Perf}(D) \arrow[r, "\simeq"] \arrow[d, hook] & \mathcal{F}(T^0) \arrow[d, hook] \\ D^b(\mathcal{X}) \arrow[r, "\simeq"] & \mathcal{F}(\check{U}, w_{\mathcal{X}}) & \operatorname{Perf}(\mathcal{D}) \arrow[r, "\simeq"] & \mathcal{F}(\Sigma^0) \end{tikzcd}$$

where $T^0 = w_Y^{-1}(\{*\}), \Sigma^0 = w_{\mathcal{X}}^{-1}(\{*\})$ are the general fibers of the respective Lefschetz fibrations. Moreover, $w_{\mathcal{X}}$ is obtained from w_Y by Lefschetz stabilization.

In other words, I describe how the process of contracting a tower of spheres to a cyclic quotient singularity, which turns Y into $\mathcal{X}$, is manifested in symplectic geometry as a procedure known as Lefschetz stabilization. This operation does not change the total space of a Landau-Ginzburg model $\check{U}$, but modifies the function w_Y to another $w_{\mathcal{X}}$ which can be much more complicated.

2. Ongoing work

Currently, I am working on a project alongside Elliot Gathercole whose focus is exact symplectic manifolds W with contact boundary ∂W which admits a periodic Reeb flow. In other words, the boundary of the manifold W admits the structure of a dynamical system such that any point eventually returns to itself. For most points, it takes a certain time T for the flow to return to its origin, but for some special points, this time is shorter: it is equal to T/n, where n is an integer. In this way, the space of orbits admits the structure of an orbifold, with a singular point associated to each orbit of length which is shorter than the principal one.

Our main result concerns an algebraic invariant known as V-shaped symplectic cohomology [2]: we show that whenever the boundary W admits this periodic structure (together with some extra technical assumptions), the algebraic structure $RFH^\bullet(W)$ also admits a periodic structure, given by the class of the principal orbit. Using this periodic structure, we are able to compute the symplectic cohomology of the 4-dimensional ADE Milnor fibers over the integers, which is a novel result.

Aside from these new computations, our results have applications in the study of compactification questions. For example, we completely classify compactifications of $\mathbb{C}^3$ which have a single divisor with a single orbifold point: these are the pairs $(\mathbb{P}(1, 1, 1, k), \mathbb{P}(1, 1, k))$ for any k of two weighted projective spaces.

3. Future work and open problems

3.1. **Higher dimensional divisorial contractions.** Surface cyclic quotient singularities can be obtained by taking a tower of spheres and crushing them to a point. This operation is the lowest-dimensional instance of a general construction in algebraic geometry known as a toroidal divisorial contraction. This type of operation is of foundational importance in birational geometry. In [11] I described how this operation of contracting a chain of spheres in algebraic geometry is manifested in the world of symplectic geometry using the process known as Lefschetz stabilization. In the future, I would like to prove a theorem generalizing this to higher dimensions, which will serve as a dictionary between algebraic and symplectic geometry, interpreting divisorial contractions as Lefschetz stabilizations.

3.2. **Compactification problems.** A recent breakthrough [8] uses symplectic cohomology to classify all smooth Kahler compactifications of $\mathbb{C}^n$: these are just the pairs $\mathbb{P}^n$ minus a hyperplane $\mathbb{P}^{n-1}$. I hope that my joint work with Elliot Gathercole on the symplectic cohomology of orbifold divisor complements can be used to prove similar theorems where one allows for the compactifying divisor to be an orbifold.

References

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