RESEARCH STATEMENT (LONGER)

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Overview

I study mathematical spaces through the lens of two distinct types of geometry: algebraic and symplectic. The former has its roots in formalising Hamiltonian dynamics, which for example can describe the movement of celestial bodies. The latter, on the other hand, originated in the study of polynomial equations and their zero sets. These two types of geometry are connected by a deep and mysterious duality called mirror symmetry.

A common theme in my research is a type of singular space called an orbifold whose singularities are locally modelled on a flat space which is folded along a set of symmetries. The geometry of orbifolds is intimately related to the representation theory of finite groups. While orbifolds have historically occupied a more central place in algebraic geometry, recent advances in orbifold Floer theory and orbifold Gromov-Witten theory ([15]) have led to a surge of interest in them from the symplectic geometry community.

For a big part of my PhD, I investigated orbifolds through the viewpoint of the homological mirror symmetry (HMS) conjecture which states that certain categorical structures within algebraic and symplectic geometry are intertwined. I focused on a class of varieties called log Calabi-Yau surfaces and showed in [17] how to extend HMS from the smooth case to the orbifold case.

During my investigations into HMS for orbifolds I studied them as algebraic varieties and related them to their symplectic counterparts. However, in current work-in-progress with Elliot Gathercole, we consider orbifolds from the symplectic point of view in order to study an invariant known as symplectic cohomology. We develop new computational tools which we apply to compute the symplectic cohomology of various smoothings of hypersurface singularities. As another sample application of our results, we completely classify all compactifications of $\mathbb{C}^3$ with an orbifold divisor admitting at most one orbifold point.

1. Homological mirror symmetry for log Calabi-Yau surfaces with cyclic quotient singularities

1.1. **Singularities and divisorial contractions.** In algebraic geometry, in particular the field of birational geometry, one often studies spaces up to contractions and blowups. These are modifications of a variety in codimension 1, meaning that 'most' of the variety is still in tact, yet a part of it might have shrunk (potentially creating a singularity) or, conversely, expanded.

Question 1. *Is there a way to understand, via mirror symmetry, what these operations correspond to in terms of symplectic topology? Moreover, in the case of quotient singularities $\mathbb{C}^n/G$, what is the relationship between representations of the group G and the geometry of the mirror?*

In my paper [17], which will form part of my thesis, I answered this question positively in the case of rational projective surfaces. Namely, I described a dictionary intertwining:

- On the one hand, the operation of contracting a chain of negative curves in a rational projective surface Y. This can be done whenever the intersection matrix of the curves is negative-definite, producing a cyclic quotient surface singularity.

- On the other hand, an instance of an operation on 4-dimensional Lefschetz fibrations known as **Lefschetz stabilization**, which modifies a Lefschetz fibration $w_Y : W \to \mathbb{C}$ by changing the function w_Y to another one $w_{\mathcal{X}}$ whose fiber is more complicated and which now has more critical points. This operation is a symplectic instance of the Morse handle cancellation: a certain number of 1-handles are added, increasing the complexity of the topology of the fiber, after which a set of cancelling 2-handles are added, ensuring that the total space remains the same.

For this dictionary to be made precise, one needs to introduce certain structural invariants of the spaces in question: these are the derived category of coherent sheaves in algebraic geometry and the Fukaya-Seidel category of vanishing cycles in symplectic geometry.

1.2. **The derived category of an orbifold surface.** The first step towards answering Question 1, at least in the case of surfaces, is to understand the derived category of an orbifold surface $\mathcal{X}$. This is a certain algebraic structural gadget which contains information about all subvarieties of $\mathcal{X}$, as well as vector bundles on them. A theorem of Ishii and Ueda [9] describes these as a combination of the derived category of a smooth surface (the minimal resolution of the underlying space of an orbifold surface), together with another piece which is local to the orbifold points of $\mathcal{X}$.

Theorem 1.1. *(Ishii and Ueda) Suppose $\mathcal{X}$ is the orbifold obtained by contracting chains of rational curves on Y to cyclic quotient singularities. Then there is a fully faithful inclusion $\Phi : D^b(Y) \to D^b(\mathcal{X})$ as well as a decomposition*

$$D^b(\mathcal{X}) = \langle \mathbf{e}, \Phi D^b(Y)\rangle$$

where the support of each object in $\mathbf{e}$ is contained in the orbifold locus of $\mathcal{X}$. Moreover, $\mathbf{e}$ admits a full exceptional collection.

1.3. **Mirror symmetry for log Calabi-Yau varieties.** There is a trichotomy in algebraic geometry governed by a quantity known as the first Chern class which, roughly, measures their curvature. Of particular importance are the ones which are flat: these are the Calabi-Yau manifolds, which first gained importance in various models of supersymmetric field theory and string theory. Calabi-Yau manifolds themselves come into two types: the compact ones, which have finite volume, and the open ones, which are unbounded. Mathematically and physically, it is an important problem to compactify the unbounded Calabi-Yau manifolds, which can be thought of as filling in the gaps of infinite area, or alternatively bounding the manifold with a sort of wall called a divisor. In complex dimension two, such compactifications are called log Calabi-Yau surfaces.

The same open Calabi-Yau manifold U can have many different compactifications, some of which can have orbifold singularities. Moreover, somewhat surprisingly, most of the geometry of the compact surface is contained entirely in the compactifying divisor. The upshot is that one can understand a 2-dimensional surface by carefully studying how the compactifying divisor sits inside of it.

Mirror symmetry is a duality in mathematics and physics which exchanges Calabi-Yau manifolds: the principle is that two dual spaces, called the A-model and B-model, lead to the exact same physical theory. This duality is compatible with compactifications: if U is mirror dual to $\check{U}$ and (Y, D) compactifies $U = Y \setminus D$, then there should be an associated a function f_Y on $\check{U}$. This function defines what is called a Landau-Ginzburg model. The homological mirror symmetry conjecture predicts that mirror spaces carry the same underlying categorical structure: $D^b(Y) \simeq \mathcal{F}(W, w)$. This conjecture has been verified by Hacking and Keating [7] for smooth log Calabi-Yau surfaces, building on previous work of Gross-Hacking-Keel. Moreover, it is proved in [13] for a special class of singular spaces known as nodal stacky curves which are the compactifying divisors in an orbifold log CY pair. In my paper, I extend this conjecture to include the case of log Calabi-Yau surfaces with cyclic quotient singularities. This amounts to a reasonable jump in complexity: the mirror to a smooth log CY is a Lefschetz fibration whose fiber is a torus (genus one), whereas the mirror to an orbifold log CY might have fibers of arbitrarily high genus.

Theorem 1.2. *([17]) Suppose $(\mathcal{X}, \mathcal{D})$ is an orbifold log Calabi-Yau compactification of an open Calabi-Yau variety U, with $\operatorname{Sing}\mathcal{X} = \operatorname{Sing}\mathcal{D}$. Let (Y, D) be the minimal resolution of the coarse space of $\mathcal{X}$. Provided that Y is at the large complex structure, there are equivalences (horizontal) and fully faithful inclusions (vertical) fitting into the following diagram:*

$$\begin{array}{ccccccc} D^b(Y) & \xrightarrow{\simeq} & \mathcal{F}(\check{U}, w_Y) & \qquad & \mathrm{Perf}(D) & \xrightarrow{\simeq} & \mathcal{F}(T^0) \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ D^b(\mathcal{X}) & \xrightarrow{\simeq} & \mathcal{F}(\check{U}, w_{\mathcal{X}}) & & \mathrm{Perf}(\mathcal{D}) & \xrightarrow{\simeq} & \mathcal{F}(\Sigma^0) \end{array}$$

where $T^0 = w_Y^{-1}(\{*\}), \Sigma^0 = w_{\mathcal{X}}^{-1}(\{*\})$ are the general fibers of the respective Lefschetz fibrations. Moreover, $w_{\mathcal{X}}$ is obtained from w_Y by Lefschetz stabilization.

2. Ongoing work

Currently, I am working on a project alongside Elliot Gathercole whose focus is exact symplectic manifolds W with contact boundary ∂W which admits a periodic Reeb flow. In other words, the boundary of the manifold W admits the structure of a dynamical system such that any point eventually returns to itself. For most points, it takes a certain time T for the flow to return to its origin, but for some special points, this time is shorter: it is equal to T/n, where n is an integer. In this way, the space of orbits admits the structure of an orbifold, with a singular point associated to each orbit of length which is shorter than the principal one.

Our main result concerns an algebraic invariant known as V-shaped symplectic cohomology [4]: we show that whenever the boundary W admits this periodic structure (together with some extra technical assumptions), the algebraic structure $RFH^\bullet(W)$ also admits a periodic structure, given by the class of the principal orbit. Using this periodic structure, we are able to compute the symplectic cohomology of the 4-dimensional ADE Milnor fibers over the integers, which is a novel result.

Theorem 2.1. *(Work in progress) Suppose W is a Liouville filling of a contact manifold $Y = \partial W$ with a periodic Reeb flow. If:*

- *The class of the principal orbit M_1 is of nonzero degree*
- *The quotient orbifold $\partial W/S^1 = \mathcal{D}$ is such that $H^\bullet(I\mathcal{D})$ is concentrated in even degrees*

then M_1 defines an isomorphism

$$RFH^\bullet(W) \simeq RFH^{\bullet - 2I}(W)$$

Corollary 2.2. *Suppose W is the Milnor fiber of a quasihomogenous hypersurface singularity $f = 0$ of degree d and weights $(w_1, \ldots, w_n)$. Then, if $\sum w_i - d > 0$ and the projective orbifold $\{f = 0\} \subset \mathbb{P}(w_1, \ldots, w_n)$ satisfies the conditions of the theorem, the Rabinowitz Floer homology of W is periodic with periodicity index $2(\sum w_i - d)$.*

In particular, this theorem applies whenever the quasihomogenous polynomial defines an orbifold $\mathbb{P}^1$. For example, our result, combined with various manipulations using the long eact sequence of Rabinowitz Floer homology and symplectic cohomology, allows us to show:

Corollary 2.3. *The symplectic cohomology of the Milnor fiber of $xy + z^p = 0$ satisfies*

$$SH^2(W) = \mathbb{Z}^{p-1}, \quad SH^{1-2\bullet} = \mathbb{Z}^{p-1}, \quad SH^{-2\bullet} = \mathbb{Z}^{p-1} \oplus \mathbb{Z}/p\mathbb{Z}, \quad \bullet \leq 0$$

The symplectic cohomology of this Milnor fiber was first computed over a field in [5] using Hochschild cohomology, but our computation is purely geometric and works over the integers, where the torsion is visible.

Aside from these new computations, our results have applications in the study of compactification questions. For example, we completely classify compactifications of $\mathbb{C}^3$ which have a single divisor with a single orbifold point: these are the pairs $(\mathbb{P}(1,1,1,k), \mathbb{P}(1,1,k))$ for any k of two weighted projective spaces.

3. Future work and open problems

3.1. **Higher dimensional divisorial contractions.** The operation of contracting a tower of spheres to a point is the lowest-dimensional instance of a general procedure in algebraic geometry known as a **toroidal divisorial contraction**. Yujiro Kawamata has proved higher dimensional instances of the special McKay correspondence, which relates $D^b(Y)$ to $D^b(X)$ whenever X is obtained from Y by such a contraction.

Example 3.1. Suppose Y contains a $\mathbb{P}^{r-1}$ with normal bundle $\mathcal{O}(-n)$. This can be contracted to a $\frac{1}{n}(\underbrace{1,1,\dots,1}_{r})$ singularity. When $n > r$ there is a SOD

$$D^b(\mathcal{X}) = \langle e_1, \dots, e_{n-r}, \Phi D^b(Y)\rangle$$

In the future, I would like to prove a theorem generalizing the one in [17] to higher dimensions, which will serve as a dictionary between algebraic and symplectic geometry, interpreting divisorial contractions as Lefschetz stabilizations. Ideally, this will be phrased using the language of sectors and stops from GPS and would interpret special types of divisorial contractions as attaching cancelling collection of n and $n-1$ handles to the mirror $(W^{2n}, \mathbf{f})$.

Conjecture 3.2. *Suppose (Y, D) is a log Calabi-Yau pair and $D^b(Y) \simeq \mathcal{F}(W, \mathbf{f}_Y), \mathrm{Perf}(D) \simeq \mathcal{F}(\mathbf{f}_Y)$. Then, one can obtain the mirror to $\mathcal{X}$ as $(W^{2n}, \mathbf{f}_{\mathcal{X}})$ where*

- *The stop $\mathbf{f}_{\mathcal{X}}$ is the mirror to the anticanonical divisor $\mathcal{D}$, obtained by attaching or cutting $n-1$-handles to/from $\mathbf{f}_Y$*
- *$(W^{2n}, \mathbf{f}_{\mathcal{X}})$ is obtained by simultaneously attaching/cutting a cancelling collection of n-handles and $n-1$-handles from $(W^{2n}, \mathbf{f}_Y)$.*

3.2. **Explicit LG models for orbifold del Pezzo surfaces.** While my theorem produces a valid construction for Homological Mirror Symmetry, the function $w_{\mathcal{X}}$ uses topological techniques that are not always explicitly computable. Ideally, the function in question will be for example a Laurent polynomial in two variables. There is a program initiated by Corti-Coates-et al [1] which aims to describe such explicit functions from a completely different viewpoint, studying what are called toric degenerations of orbifold del Pezzo surfaces. These types of algebraic surfaces are a strict subset of all rational surfaces, but they are the only ones where there is hope of producing such an algorithm. In the future, I would like to understand better the relationship between the functions I define using symplectic topology and the Laurent polynomials from [1].

As a stepping stone, in my paper I compared my model $w_{\mathcal{X}}$ to an explicit function f arising from a toric degeneration in the case of an infinite family of spaces X_{k+1} which are hypersurfaces in weighted projective space. After hefty computation, I showed that moving some of the coefficient parameters in f close to zero separates the critical values of f in a particularly pleasant way, analogous to the separation of $\Phi D^b(Y)$ and $\mathbf{e}$ on the B-side.

3.2.1. *Surfaces with empty anticanonical linear system.* Another interesting question initiated by Gugiatti and Rota [6] is whether Homological Mirror Symmetry still holds for certain special surfaces which have empty anticanonical linear system - for these, standard mirror symmetry techniques tend to fail, as they don't admit toric degenerations. However, in some cases, such surfaces are 'close' to others which fall within the scope of my theorem. One approach is to use a tool known as deformation theory, which allows us to pass from a surface to ones nearby it via a deformation of complex structures. On the symplectic side, this deformation is encoded in an element of a special group denoted $SH^2(W, w_{\mathcal{X}})$. Finally, it would be interesting to relate this algebraic deformation to one defined geometrically by counting holomorphic strips with punctures asymptotic to a specific Reeb orbit.

4. Dubrovin's conjecture

Dubrovin's conjecture states that a variety admits a full exceptional collection if and only if its quantum cohomology is generically semisimple. This has been verified in the case of smooth rational surfaces, which are blowups of $\mathbb{P}^2$. However, by the theorem of Ishii and Ueda, orbifold rational surfaces also admit full exceptional collections and it is an open question to show that their quantum rings are semisimple. A potential way forward is to use a method akin to Iritani's blowup formula [8], which has now been generalized for more general VGIT: in other words, to first show that there is some change of coordinates realizing $QH^\bullet(Y)$ as a summand of $QH^\bullet(\mathcal{X})$ and then to understand the remaining component.

4.1. **Compactification problems.** A recent breakthrough [14] uses symplectic cohomology to classify all smooth Kahler compactifications of $\mathbb{C}^n$: these are just the pairs $\mathbb{P}^n$ minus a hyperplane $\mathbb{P}^{n-1}$. I hope that my joint work with Elliot Gathercole on the symplectic cohomology of orbifold divisor complements can be used to prove similar theorems where one allows for the compactifying divisor to be an orbifold.

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